



Poznan University of Technology  
Division of Virtual Engineering



Michał Nowak

Simultaneous Size, Shape and Topology Optimization in Parallel Numerical  
Environment

Cosmoprojector

# The Stiffest Design

(Energetic condition on the surface)

Following the optimal design discussion (Pedersen [2003](#)):

defining the total potential  $\Pi$  as a sum of elastic energy and work of external forces:

$$\Pi = U_{\tau} + U_{\text{ext}} \quad (1)$$

assuming design-independent external loads, with the respect for virtual work principle:

$$\frac{\partial \Pi}{\partial \varepsilon} = 0 \quad (2)$$

the derivative of the total potential with respect to an arbitrary parameter  $h$  can be written as follows:

$$\frac{d\Pi}{dh} = \frac{\partial U_{\tau}}{\partial h}. \quad (3)$$

## The Stiffest Design cont. (Energetic condition on the surface)

for the design-independent external loads, and for the local design parameter  $h_e$  in the domain  $e$ , that changes the design in the domain only, the following formula can be employed (localized determination of the sensitivity for the total elastic strain energy):

$$\frac{dU_\epsilon}{dh_e} = - \left( \frac{\delta((\bar{u}_\epsilon)_e V_e)}{\delta h_e} \right)_{fixed\ strains} \quad (4)$$

for all parameters in case of optimization for extremum elastic strain energy, together with the assumption of constant total volume  $V$  of the structure the increment of the objective corresponding to parameter increments:

$$\Delta U_\epsilon = \sum_e u_\epsilon \frac{dV_e}{dh_e} \Delta h_e \quad (5)$$

when in turn, we take into account the necessary condition for optimality:

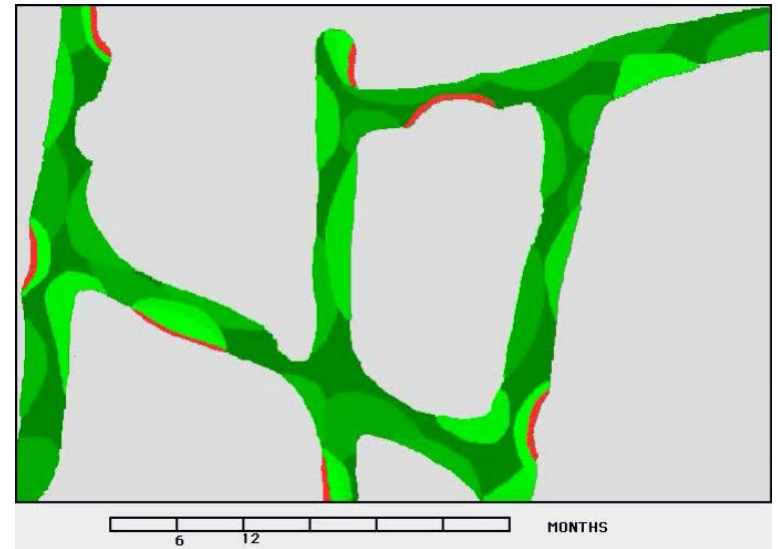
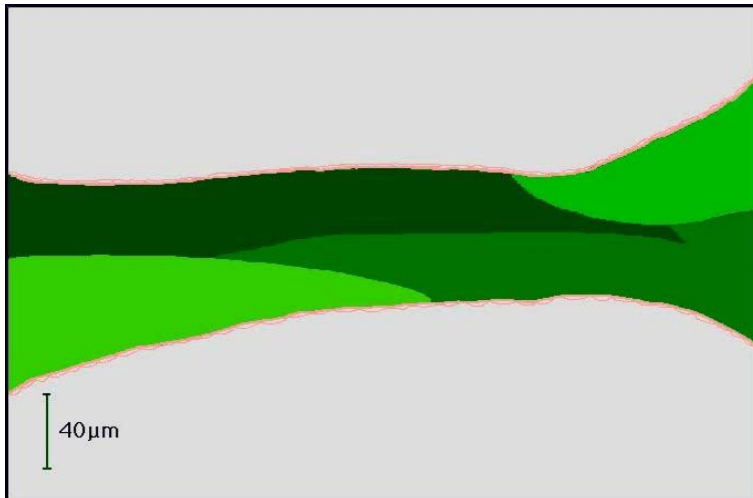
$$\Delta U_\epsilon = 0 \quad (6)$$

we can conclude, that for the stiffest design the strain energy density along the shape to be designed must be constant:

$$u_{\epsilon_s} = const. \quad (7)$$

The optimization goal can also be formulated as a minimum volume problem with assumed fixed strain energy, as described in Dzieniszewski (1983). The resulting condition concerning the SED is the same as the case of minimum compliance, thus the value of the SED on the designed surface must be equal when the volume is minimal by the assumed value of the strain energy in the structure.

# Biomimetic Optimization Technology - Trabecular Bone Remodeling Phenomenon



<http://courses.washington.edu/bonephys/>

# Biomimetic Optimization Technology - Trabecular Bone Remodeling Phenomenon cont.

**'Mechanosensitivity'**

**On the surface only!**

Huiskes Ruimerman

'Regulatory model'

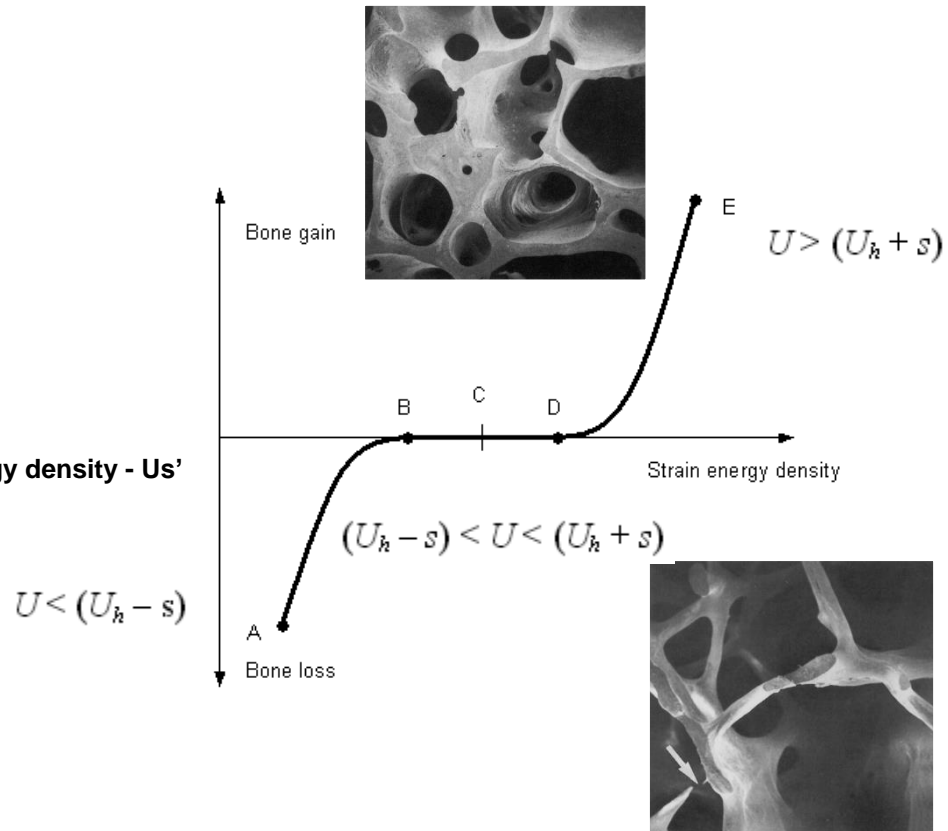
(Huiskes et al. [2003](#))

**'Homeostatic strain energy density -  $U_s$ '**

Perfect balance between resorption and new tissue creation.

'Lazy zone'

Carter (Carter et al. [1989](#))



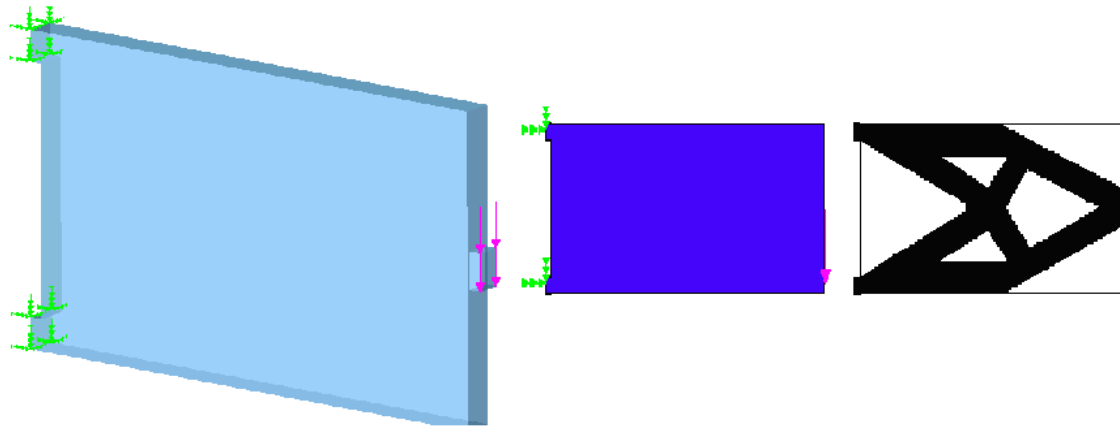
# Comparison to the Topology Optimization

**‘Mechanosensitivity’ – SED distribution from the FEM model**

**Structural evolution - on the surface only!**

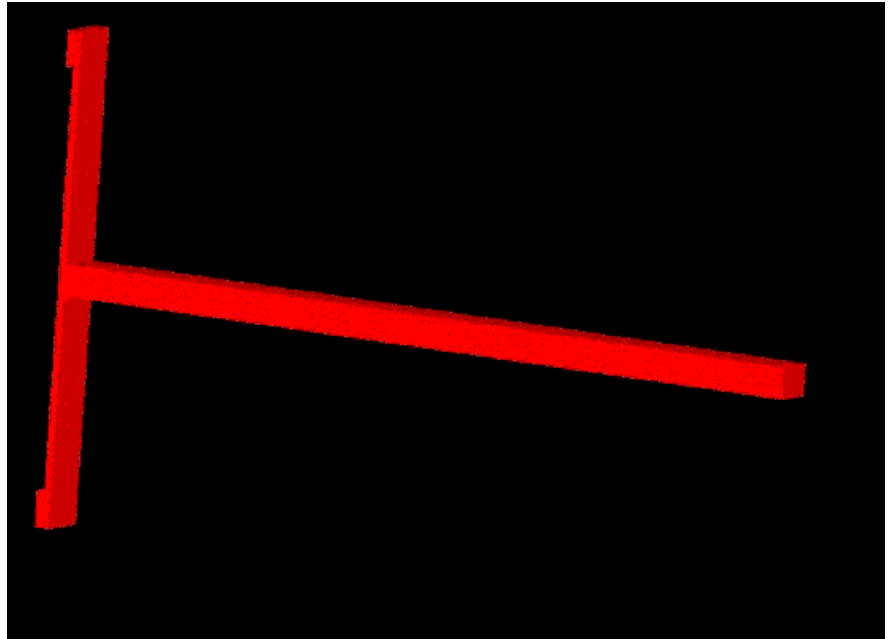
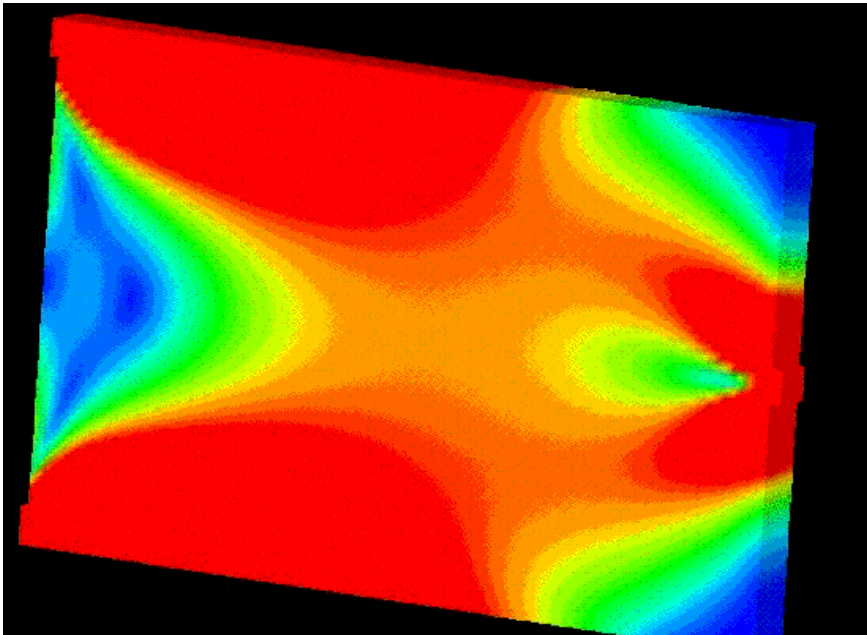
Cantilever beam  
example

Bendsoe M.P., and Sigmund O.  
Topology Optimization - Theory, Methods and Applications  
ISBN 3-540-42992-1, Springer Verlag, 2003.



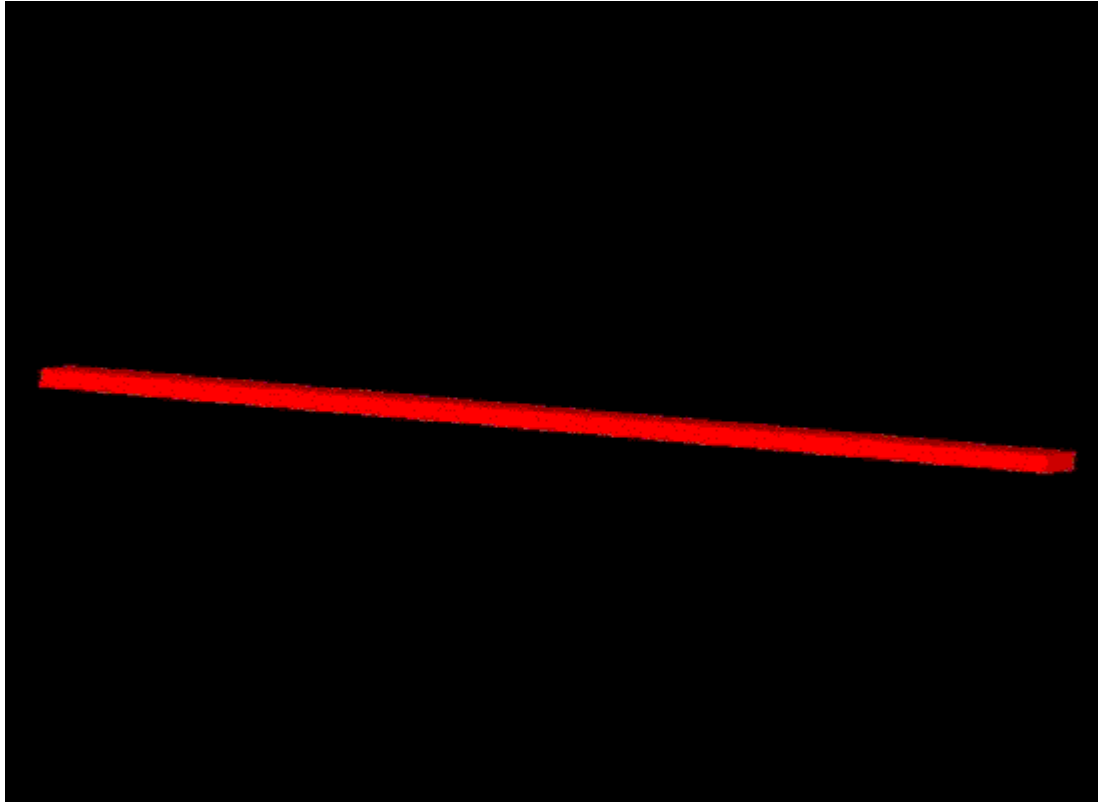
# Comparison to the Topology Optimization

Cantilever beam  
example



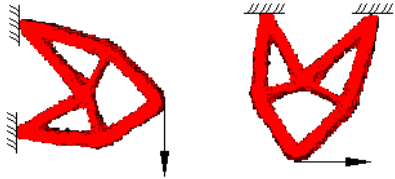
# Comparison to the Topology Optimization

Clamped wall  
bending force

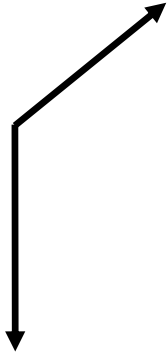
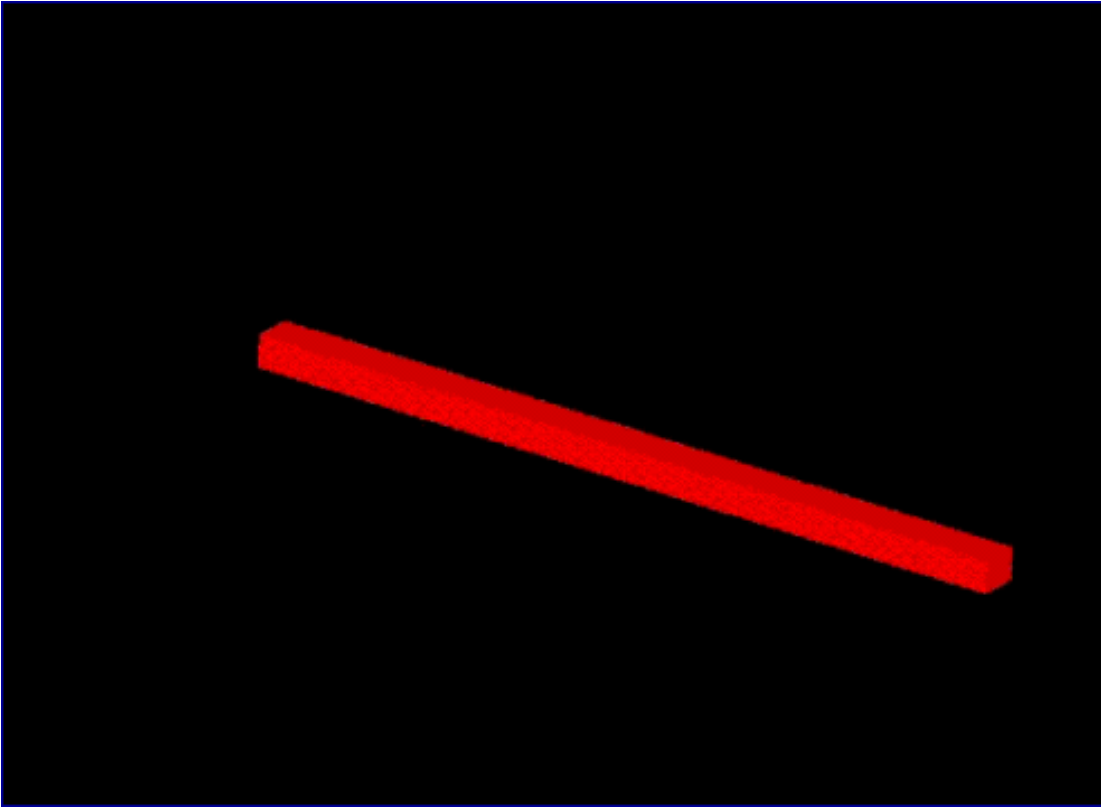




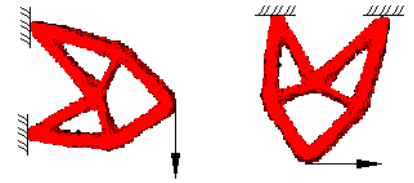
# Multiple load case example



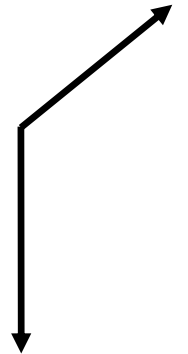
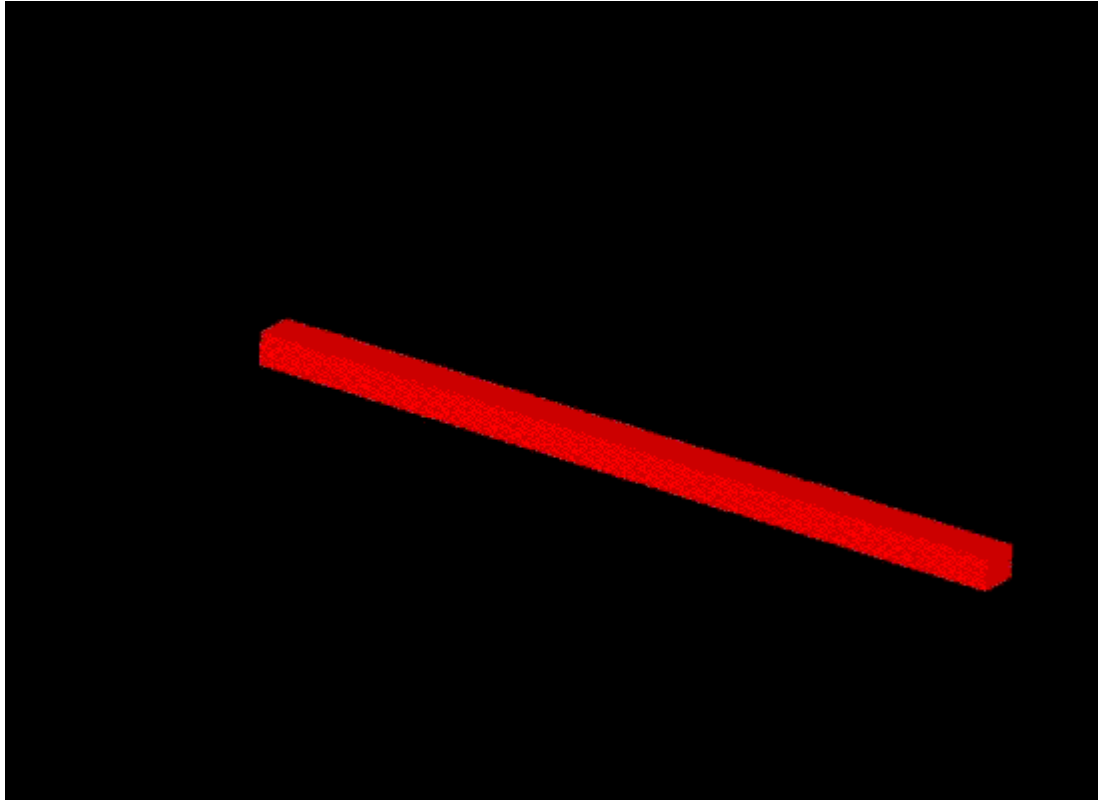
Clamped wall  
bending force



# Multiple load case example



Clamped wall  
bending force





Poznan University of Technology  
Division of Virtual Engineering

Dr Michal Nowak

ul.Piotrowo 3  
60-965 Poznan, Poland

tel. (+4861) 665-2041

fax. (+4861) 665-2618

<http://stanton.ice.put.poznan.pl/nowak>

e-mail: [Michal.Nowak@put.poznan.pl](mailto:Michal.Nowak@put.poznan.pl)

Cosmoprojector

Biomimetic Topology Optimization Method